BASIC PROPERTIES OF THE MODEL:

The analysis of the model system will provide us with the understanding of the dynamics of TB with proper controls. The model equation (1) to (6) are said to be epidemiologically realistic and mathematically well-posed in the domain.(=(,0,0,0, ,0)

The tub population S,E,I,are all non-negative and have lumps less than or equal to Λ⁄U,which implies that the model equations are valid epidemically.

POSITIVITY OF SOLUTION:

Lemma1: We claim and show that all the solutions of the equation (1) to (6) are positive for all time t ≥ 0 provided that the initial conditions are positives. By which et al (2014), le),t {(sc0),Ec0l,Ic0),col,c0l,Rc0l)≥0}E= =˄+R-(µ+(1-µ)T)s clearly-(µ+(1-µ) T)s ≥ -(µ+(1-µ)t)dt lns≥-(µ+(1-µl0)t+c. ≥t+c ;k,=

Now at t=0,s(0)=Sos(0)≥S(t)≥S(0) OR [(µ+(1-µ)T)S

-[(1-µ)(S Where α= . Now, write as, , so that S’≥-[(1-µ)(. S(t)≥ S(0) [(1-µ)( -µt S(t)≥S(0)

S(t)≥s(0) Is the face of injection where are the contact rates in the I, components respectively. From the second equation; =(1-µ) Clearly =+(]E. ≥[E where = -[. -[] E(t) E

(0) ≥ 0  E(t)E(0)+µ] t ≥ 0.

From equation 3: = (1-PE-[k++E+(1-)µ]T  -(+ )I

Where =K+E+(1- )  -  -  I (t)  I(0) . From equation 4; ==(I- ) -  I+ –() of cause, )  -( + +T+ ) - (K5+ ) (t)  (0) 0 R(t) R(0) t 0.

FEASIBLE REGION

Lemma 2: The region; {(SCO), I, (0), col ,Rcol) 0} E is positively-invariant and all solution contain in this region. Proof: Let {(scol ,Eco ),Ico),(v),co),R(0)} any solution of the system with positive initial conditions. Summing equation (1) to (6) gives = =UN-(.) At the disease free state, I=== O and we have -  +µN.the integrating factor I.f. will be = .therefore, +.Applying initial and conditions, we have t=0,N(0) = =- . Here N(t) +(- ). With the application if Birkoff and Rita’s there on differential inequality (Birkoff, 1982) , we obtain 0 N as t This establishes the fact that the total population approves . Thus, the feasible solution set of this model enters the region [{(scol ,Eco ),Ico),col, 0] E . this suffices that Mr. model is epidemiologically valid and meaningful and concludes that the model is mathematically well posed. BIFURCATION ANALYSIS:

Recall that : - - (kl+xl+ᶴ+{ ) but at the disease free state I= = Let S = E = I= = R = . N = . Re-write as = = + - since (1-u)T –[ )] = =(1- ) = =() + ᶴ+ + ) = =E+–(++ҿ+]) = =z+ y+ –( 4+4+ . The Jacobian of the transformed system evaluated at the DFE () with 0= is given by; =J(\ =ð

Clearly has negative real parts as its Eigen.

Global sterility of Disease free . Equilibrium State: Proof: The two conditions ) and ) in Castillo – Chavez global stability theorem must be satisfied for to establish the global stability of the disease free equilibrium. Write the model equation (1) to (6) in the From: = F (X,) = G (X,Z); G (X,0)=0 With components X=(and Z=( , ) where X E denotes unifeduc population and ZE1denotes the injected population. The desease free equilibrium is denoted by =(0) where =(,0). For condition (the global systematic stability of to be satisfied we have =F(x,0)= where= That is, = and –(R = . Solving the differencial equations any method of integrating factor l.f, we obtain the following: += +. I.f == s(t)= ᶴ (+ C s(t)= (+ C . Applying initial value t=0, S(0)= we get S(0) ( C = S(0)- (s(t)=(+ S(0)- (S(t)=(1- (+ S(0)Again = - M5R =Mt5 + C R(t)= KNow at t=0, R(0) = =K R (t) =(0) It is obvious that (t) and (t). Thus,= (i.e. globally asymptotically stable. From G(X,Z) = ==Now, A= ( = It is easy to see that the off diagonal elements are non-negative elements so that (X,Z) = AZ – G (X,Z) = Hence, = (,0) is globally asymptotically stable. This establishes the fact that no matter the number of injected individuals in the population under review, TB can be controlled.

LOCAL STALILITY OF ENDEMIC EQUILIBBRIUM STATE.

Rename equation as (S,E,I, = (. Let us make use of vector notation thus: X = ( and y – write equation ((1) to (6) as = F( . Thus = + 2x6 – ( (1-P). ==(1-E+(1-u-x].=(1-u+Y-(ᶴ+u+ψ). = E+ -(y+ +z+ ). =z +y +-(2+4+u). And . The Jacobian of these model equations are the desease free state is now given by J=Let us choose =as Mr. bifurcation parameter so that we will show weather the model system exhibits a backward or forward bifurcation at = (,Gumel and song, α00s these implies that VJ( (solving for , we obtain the following:u =0 ……………..(1) - + + ……….(2) + + =0………(3) + + + =0………(4) + +y - =0……………..(5) + - ………………….(6)=From equation (6) =0 - = ……..(7) substitute into (2) to get (+ + =0 [] + =0 =[ - = ………(8) substitute into (3) to get ( - + +E =0 [ + u + E =0 Eliminate From (4) and (5) then (4) x + + + =0 (1) (5) + + y- + =0 (11) Add (10) to get